

33.9 Crude oil is pumped by a centrifugal pump with a performance curve given by the equation $H_p = 50 - Q - \frac{Q^2}{25}$, where head is in units of feet and flow rate is in units of cubic feet per second. The system curve is described by the equation $H_s = \frac{Q^2}{25} + \frac{Q}{10}$, with the same units. Neglecting efficiency and losses, what is the power required by the pump?

- A. 16hp
- B. 30hp
- C. 35hp
- D. 53hp

The intersection between the performance curve for a given pump and the system curve is the operating point. By setting the equations equal, the volume flow rate and head pressure at the operating point can be determined, and then used to determine the power.

$$H_p = H_s$$

$$50 - Q - \frac{Q^2}{25} = \frac{Q^2}{25} + \frac{Q}{10}$$

$$\frac{2Q^2}{25} + \frac{11Q}{10} - 50 = 0$$

Multiply by 50 to eliminate fractions, then solve the quadratic equation.

$$4Q^2 + 55Q - 2500 = 0$$

Recall there will be two possible roots taking the form: $Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 4$, $b = 55$, $c = -2500$

$$\sqrt{b^2 - 4ac} = \sqrt{(55)^2 - 4(4)(-2500)} = \sqrt{43,025} \approx 207.42$$

Note that one of the roots taking the form $Q = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ will have a negative value. This is mathematically valid but practically not reasonable, so that solution should be discarded. The root with a positive value is the volume flow rate for the operating point. Solve and convert to *gpm*.

$$Q = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-55 + 207.42}{2(4)} = 19.05 \frac{ft^3}{s} \left(\frac{7.48gal}{1ft^3} \right) \left(\frac{60s}{1min} \right) = 8550gpm$$

Substitute back into either of the equations to find the corresponding head pressure at the operating point:

$$H_s = \frac{Q^2}{25} + \frac{Q}{10} = \frac{(19.05)^2}{25} + \frac{19.05}{10} = 16.43ft$$

Note the head pressure is given in units of feet; however, it is feet of **Crude Oil**—not water. Therefore the **Specific Gravity**, must be accounted for when the power is determined. Also note the efficiency is to be neglected, thus $bhp \sim whp$.

$$bhp = whp = \frac{Q_{[gpm]} \Delta h_{[ft]} \cdot SG}{3960} = \frac{(8550)(16.43)(0.86)}{3960} = 30.5hp$$

Answer B